

$f(\mathcal{R})$ -Einstein-Palatini Formalism and smooth branes

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In this work, we present the $f(\mathcal{R})$ -Einstein-Palatini formalism in arbitrary dimensions and the study of consistency applied to brane models, the so-called braneworld sum rules. We show that it is possible a scenario of thick branes in five dimensions with compact extra dimension in the framework of the $f(R)$ -Einstein-Palatini theory by the accomplishment of an assertive criteria.

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I. INTRODUCTION

Braneworld models have received a large amount of attention in the high-energy community since the outstanding Randall-Sundrum model [1], providing a precise relation between a warped geometry and the mass scale of an effective TeV universe. Soon after the establishment of warped models, a plethora of models, generalizations, and applications were developed [2]. Most importantly to our purposes was the smooth extension of warped branes, first introduced by Gremm in [3]. From the perspective that there must exist a typical length scale below what our understanding of the physical laws should be, at least, superseded by a full quantum gravity theory, the idea of infinitely thin branes, as used in the Randall-Sundrum model, is only an approximation, though highly nontrivial.

A crucial point concerning smooth extensions of braneworlds (see [4] for a comprehensive review) in General Relativity theory is that it is always necessary to preclude of the extra dimension orbifold topology used in the original Randall-Sundrum model. After all, the S^1/\mathbb{Z}_2 is also important to make contact to Hořava-Witten theory [5]. With effect, there is an exhaustive theorem which forbids smooth generalizations of the usual Randall-Sundrum model [6] (see also [7]). By usual, we mean a five dimensional braneworld endowed with non-separable geometry whose extra dimensions are compact, within the context of General Relativity. In a gravitational theory different from General Relativity, however, the situation may be different. In fact, by applying the so-called braneworld

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sum rules (a set of consistency conditions obtained from the gravitational equations of motion) it is possible to see that smooth generalizations of the Randall-Sundrum framework are indeed possible.

Many extensions based upon the ideas delineated in the previous paragraph was done. The investigation of braneworld sum rules applied to smooth branes generalization in the context of Brans-Dicke and $f(\mathcal{R})$ gravity has been studied in some detail in [8]. In these cases it is always possible to show that the sum rules can be relaxed by the presence of additional terms coming from the gravitational theory (other than the usual case) in question. The $f(\mathcal{R})$ theory analyzed in [8] was worked out in the light of the metric formalism. As it is well known, however, that the metric and Palatini formalisms are not equivalent in the approach to $f(\mathcal{R})$ gravity [9, 10]. One of the main differences between the two approaches is given by the fact that in the metric formalism the trace of fields equations gives rise to a dynamical degree of freedom, whilst in the Palatini formalism this procedure lead to an algebraic constraint. Concerning the problem we are interested here, we shall see that the necessary condition leading to a smooth brane extension is considerably modified, presenting a more clear criteria, namely, a $f(\mathcal{R})$ theory with negative first derivative with respect to R .

One of the difficulties that may arise when performing the sum rules regards the Einstein tensor managing. Here, we use a simple conform transformation in the metric to accomplish the sum rules program and extend the consistency conditions in the scenario of theories $f(\mathcal{R})$ adopting the Palatini formalism. This work is organized as follows. In Section II, we briefly present the sum rules idea for braneworld scenarios. In the following Section we construct the field equations in a $f(\mathcal{R})$ -Einstein-Palatini formalism in arbitrary dimensions. In Section IV, we apply the sum rules to the $f(\mathcal{R})$ -Einstein-Palatini case, investigating the relevant condition which leads to smooth braneworlds. In the last Section we conclude.

II. SUM RULES FOR BRANEWORLD SCENARIOS

Much of the necessary formalism to the implementation of sum rules in the $f(\mathcal{R})$ -Einstein-Palatini context was developed elsewhere [7, 8]. Therefore, we shall pinpoint some important aspects in this Section. By considering the spacetime as a D -dimensional manifold endowed with a non-factorisable geometry, we write the line element as

$$ds^2 = g_{AB}(X)dX^A dX^B = W^2(r)g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ab}(r)dr^a dr^b, \quad (1)$$

where $W^2(r)$ is the warp factor, X^A denotes the coordinates of the full D -dimensional spacetime, x^μ stands for the $(p+1)$ coordinates of the non-compact spacetime (brane), and r^a labels the $(D-p-1)$ directions in the internal compact space. The classical action takes into account the spacetime dynamics coupled to a scalar field, namely

$$S = S_{gravity} + \int d^D X \sqrt{-g} \left(-\frac{1}{2} \partial_A \Phi \partial^A \Phi - V(\Phi) \right), \quad (2)$$

where we assume that the scalar field has only dependence on the internal space coordinates $\Phi = \Phi(r^m)$. The scalar field above shall be understood as the responsible to generate the brane. We leave the potential unspecified since it will not be relevant in our case. The energy-momentum tensor gives

$$T_{\mu\nu} = -W^2 g_{\mu\nu} \left(\frac{1}{2} \nabla \Phi \cdot \nabla \Phi + V(\Phi) \right), \quad (3)$$

and

$$T_{ab} = \nabla_a \Phi \nabla_b \Phi - g_{ab} \left(\frac{1}{2} \nabla \Phi \cdot \nabla \Phi + V(\Phi) \right). \quad (4)$$

It is possible to show [6, 7] that the following expression holds

$$\nabla \cdot (W^\alpha \nabla W) = \frac{W^{\alpha+1}}{p(p+1)} \left[\alpha (W^{-2} \bar{R} - R_\mu^\mu) + (p-\alpha) (\tilde{R} - R_a^a) \right], \quad (5)$$

where $R_\mu^\mu = W^{-2} g^{\mu\nu} R_{\mu\nu}$ and $R_a^a = g^{ab} R_{ab}$ are the partial traces such that $R = R_\mu^\mu + R_a^a$ and α is an arbitrary parameter. Moreover, \bar{R} is the scalar of curvature derived from $g_{\mu\nu}$ and \tilde{R} the scalar of curvature associated to the internal space. The braneworld sum rules can be obtained from two considerations, one physical and one mathematical. From the physical point of view it is necessary to specify the gravitational theory in question, i. e. write $S_{gravity}$. This being done (notice that the dynamics is specified accordingly), one is able to use the fact that, as far as the internal space is periodic without boundary, the left hand side of (5) vanish under integration.

III. THE $f(R)$ -EINSTEIN-PALATINI FORMALISM IN ARBITRARY DIMENSIONS

In the so-called Palatini formalism the metric and the connection are assumed to be independent variables. The field equations are derived from the variation of the Einstein-Hilbert action with respect to metric and connection independently. Thus, the Ricci and Riemann tensors are objects constructed from a general affine connection, but without the torsion terms.

It is well known that the definition $T_{AB} \equiv 2/\sqrt{-g}\delta S_M/\delta g^{AB}$ when implemented along with the principle of least action for $f(\mathcal{R})$ -Einstein-Palatini gravity, leads to the following field equations

$$f'(\mathcal{R})\mathcal{R}_{AB} - \frac{1}{2}f(\mathcal{R})g_{AB} = 8\pi G_D T_{AB}, \quad (6)$$

and

$$-\bar{\nabla}_C(\sqrt{-g}f'(\mathcal{R})g^{AB}) + \bar{\nabla}_D(\sqrt{-g}f'(\mathcal{R})g^{D(A})\delta_C^{B)}) = 0, \quad (7)$$

such that when $f(R) = R$, the Palatini formalism restores general relativity. Rewriting Eq. (7) we get

$$-\bar{\nabla}_C(\sqrt{-g}f'(\mathcal{R})g^{AB}) + \frac{1}{2} \left[\bar{\nabla}_D(\sqrt{-g}f'(\mathcal{R})g^{DA})\delta_C^B + \bar{\nabla}_D(\sqrt{-g}f'(\mathcal{R})g^{DB})\delta_C^A \right] = 0, \quad (8)$$

and contracting the indices C e B we are left with

$$\bar{\nabla}_D(\sqrt{-g}f'(\mathcal{R})g^{DA}) = 0. \quad (9)$$

Therefore Eq. (7) reads simply

$$\bar{\nabla}_C(\sqrt{-g}f'(\mathcal{R})g^{AB}) = 0. \quad (10)$$

In this vein, by defining a metric h_{AB} as

$$h_{AB} \equiv f'(\mathcal{R})^{\frac{2}{D-2}}g_{AB}, \quad h^{AB} \equiv f'(\mathcal{R})^{\frac{2}{2-D}}g^{AB}, \quad (11)$$

we formally have the connection equation

$$\bar{\nabla}_C(\sqrt{-h}h^{AB}) = 0. \quad (12)$$

Following this clue it is possible to write the connection as

$$\bar{\Gamma}_{AB}^C = \{_{AB}^C\} + \frac{1}{2f'(\mathcal{R})^{\frac{2}{D-2}}} \Delta_{AB}^C, \quad (13)$$

where

$$\Delta_{AB}^C = \left\{ \delta_B^C \partial_A f'(\mathcal{R})^{\frac{2}{D-2}} + \delta_A^C \partial_B f'(\mathcal{R})^{\frac{2}{D-2}} - g_{AB} g^{CD} \partial_D f'(\mathcal{R})^{\frac{2}{D-2}} \right\}, \quad (14)$$

and $\{_{AB}^C\}$ are the usual Christoffel symbols.

The Ricci tensor, generalized via the conformal (11) relation, is given by $\mathcal{R}_{AB} = \partial_C \bar{\Gamma}_{AB}^C - \partial_B \bar{\Gamma}_{AC}^C + \bar{\Gamma}_{CE}^C \bar{\Gamma}_{AB}^E - \bar{\Gamma}_{BE}^C \bar{\Gamma}_{AC}^E$, and can be recast as

$$\mathcal{R}_{AB} = R_{AB} + \frac{[D-1]}{2} \frac{(\nabla_A f'(\mathcal{R})^{\frac{2}{D-2}})(\nabla_B f'(\mathcal{R})^{\frac{2}{D-2}})}{f'(\mathcal{R})^{\frac{4}{D-2}}} - \frac{1}{f'(\mathcal{R})^{\frac{2}{D-2}}} \left(\nabla_A \nabla_B + \frac{1}{2} g_{AB} \square \right) f'(\mathcal{R})^{\frac{2}{D-2}}, \quad (15)$$

and thus the generalized scalar of curvature reads

$$\mathcal{R} = R + \frac{[D-1]}{2} \frac{1}{f'(\mathcal{R})^{\frac{4}{D-2}}} \left(\nabla_A f'(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla^A f'(\mathcal{R})^{\frac{2}{D-2}} \right) - \frac{1}{f'(\mathcal{R})^{\frac{2}{D-2}}} \left(\frac{D}{2} + 1 \right) \square f'(\mathcal{R})^{\frac{2}{D-2}}. \quad (16)$$

In the Palatini formalism the field equations are given by

$$\mathcal{R}_{AB} - \frac{f}{2f'(\mathcal{R})} g_{AB} = \frac{8\pi G_D T_{AB}}{f'(\mathcal{R})}. \quad (17)$$

Hence, inserting equation (15) in (17) and adding on both sides of the term $-g_{AB}R/2$ we obtain, after some manipulation, the Einstein-Palatini field equations in arbitrary dimensions

$$\begin{aligned} R_{AB} - \frac{1}{2} R g_{AB} = & \frac{8\pi G_D T_{AB}}{F(f'(\mathcal{R}))} - \frac{g_{AB}}{2} \left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{1}{F(\mathcal{R})^{\frac{2}{D-2}}} (\nabla_A \nabla_B - g_{AB} \square) F(\mathcal{R})^{\frac{2}{D-2}} \\ & - \frac{[D-1]}{2F(\mathcal{R})^{\frac{4}{D-2}}} \left[\left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla_B F(\mathcal{R})^{\frac{2}{D-2}} \right) - \frac{g_{AB}}{2} \nabla_C F(\mathcal{R})^{\frac{2}{D-2}} \nabla^C F(\mathcal{R})^{\frac{2}{D-2}} \right]. \end{aligned} \quad (18)$$

where $F(\mathcal{R}) = df(\mathcal{R})/d\mathcal{R}$ and \mathcal{R} is Ricci scalar constructed out from \mathcal{R}_{AB} . Now we are able to implement the relevant partial traces, derived from (18), into Eq. (5).

IV. BRANEWORD SUM RULES IN $f(R)$ -EINSTEIN-PALATINI

Taking advantage of Eq. (18) we see that the scalar of curvature reads

$$\begin{aligned} R = & \frac{2}{(2-D)} \left\{ \frac{8\pi G_D}{F(\mathcal{R})} T - \frac{D}{2} \left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) - (D-1) \frac{1}{F(\mathcal{R})^{\frac{2}{D-2}}} \square F(\mathcal{R})^{\frac{2}{D-2}} \right. \\ & \left. - \frac{[D-1]}{2F(\mathcal{R})^{\frac{4}{D-2}}} \left[\left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla^A F(\mathcal{R})^{\frac{2}{D-2}} \right) - \frac{D}{2} \nabla_C F(\mathcal{R})^{\frac{2}{D-2}} \nabla^C F(\mathcal{R})^{\frac{2}{D-2}} \right] \right\}, \end{aligned} \quad (19)$$

from which, reinserting it back in (18), we have

$$\begin{aligned} R_{AB} = & \frac{1}{F(\mathcal{R})} \left[8\pi G_D \left(T_{AB} - \frac{g_{AB}}{(D-2)} T \right) \right] + \frac{\nabla_A \nabla_B F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}} + \frac{g_{AB}}{(D-2)} \left\{ \left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{\square F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}} \right. \\ & \left. - \frac{\nabla_C F(\mathcal{R})^{\frac{2}{D-2}} \nabla^C F(\mathcal{R})^{\frac{2}{D-2}}}{2F(\mathcal{R})^{\frac{4}{D-2}}} \right\} - \frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}} \left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla_B F(\mathcal{R})^{\frac{2}{D-2}} \right). \end{aligned} \quad (20)$$

The partial trace of the above equation with respect to the brane, non-compact, dimensions is given by

$$\begin{aligned} R^\mu_\mu = & \frac{1}{(D-2)F(\mathcal{R})} \left[8\pi G_D \left((D-p-3)T^\mu_\mu - (p+1)T^a_a \right) \right] + \frac{(D+p-1)}{(D-2)F(\mathcal{R})^{\frac{2}{D-2}}} (W^{-2} \nabla_\mu \nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}) \\ & + \frac{(p+1)}{(D-2)} \left[\left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{\nabla_a \nabla^a F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}} - \frac{1}{2F(\mathcal{R})^{\frac{4}{D-2}}} \left((W^{-2} \nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}} \nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}} \right. \right. \\ & \left. \left. + \nabla_c F(\mathcal{R})^{\frac{2}{D-2}} \nabla^c F(\mathcal{R})^{\frac{2}{D-2}} \right) \right] - \frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}} W^{-2} \left(\nabla_\mu F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}} \right), \end{aligned} \quad (21)$$

while its internal space counterpart reads

$$\begin{aligned}
R_a^a = & \frac{1}{(D-2)F(\mathcal{R})} \left[8\pi G_D \left((p-1)T_a^a - (D-p-1)T_\mu^\mu \right) \right] + \frac{(2D-p-3)}{F(\mathcal{R})^{\frac{2}{D-2}}(D-2)} (W^{-2}\nabla_\mu\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}) \\
& + \frac{(D-p-1)}{(D-2)} \left[\left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{(\nabla_a\nabla^a F(\mathcal{R})^{\frac{2}{D-2}})}{F(\mathcal{R})^{\frac{2}{D-2}}} - \frac{1}{2F(\mathcal{R})^{\frac{4}{D-2}}} (W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}} \right. \\
& \left. + \nabla_c F(\mathcal{R})^{\frac{2}{D-2}}\nabla^c F(\mathcal{R})^{\frac{2}{D-2}} \right] - \frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}} \left(\nabla_a F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla^a F(\mathcal{R})^{\frac{2}{D-2}} \right). \quad (22)
\end{aligned}$$

Now, by inserting (21) and (22) into equation (5), one arrives at

$$\begin{aligned}
\nabla \cdot (W^\alpha \nabla W) = & \frac{W^{\alpha+1}}{p(p+1)(D-2)F(\mathcal{R})} \left\{ 8\pi G_D \left((p-\alpha)(D-p-1) - \alpha(D-p-3) \right) T_\mu^\mu + \right. \\
& + 8\pi G_D \left(\alpha(p+1) - (p-\alpha)(p-1) \right) T_a^a + (D-2) \left(\alpha W^{-2}\bar{R} + (p-\alpha)\bar{R} \right) F(\mathcal{R}) - \\
& - \frac{W^{\alpha+1}}{p(p+1)(D-2)} \left\{ \left[\frac{W^{-2}\nabla_\mu\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}} \right] [\alpha(D+p-1) + (p-\alpha)(2D-p-3)] \right. \\
& - [\alpha(p+1) + (p-\alpha)(D-p-1)] \left[\left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{(\nabla_a\nabla^a F(\mathcal{R})^{\frac{2}{D-2}})}{F(\mathcal{R})^{\frac{2}{D-2}}} \right. \\
& \left. \left. - \frac{1}{F(\mathcal{R})^{\frac{4}{D-2}}} (W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}} + \nabla_c F(\mathcal{R})^{\frac{2}{D-2}}\nabla^c F(\mathcal{R})^{\frac{2}{D-2}}) \right] \right. \\
& + \left[\frac{(D-1)(D-3)}{2F(\mathcal{R})^{\frac{4}{D-2}}} \right] \left[\alpha(W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}}) + (p-\alpha) \right. \\
& \left. \left. \times \left(\nabla_a F(\mathcal{R})^{\frac{2}{D-2}} \right) \left(\nabla^a F(\mathcal{R})^{\frac{2}{D-2}} \right) \right] \right\}. \quad (23)
\end{aligned}$$

As a last step, by assuming the internal space compact (as in the standard cases) the left hand side of Eq. (23) vanishes upon integration. Following the standard presentation we denote these integrations by $\oint \nabla \cdot (W^\alpha \nabla W) = 0$. Hence, by inserting the energy-momentum partial traces and integrating over the internal space, it is possible to obtain the sum rules to the very general case in the scope of $f(\mathcal{R})$ -Einstein-Palatini theory. The result is quite large, and its generality contributes to overshadow its physical content.

In order to extract physical information it is convenient to particularize the analysis to the $D = 5$ and $p = 3$ case. Thus we shall investigate a five-dimensional bulk with an unique extra dimension ($\tilde{R} = 0$) endowed to a orbifold topology, for instance. Besides, in this four-dimensional brane context we can implement the physical constraint ($\bar{R} = 0$) in trying to describe our universe in large scales. Therefore, after these particularizations, and using equations (3) and (4) for the

sources, we have the following set of conditions

$$\begin{aligned}
0 = & 8\pi G_5 \oint \frac{W^{\alpha+1}}{F(\mathcal{R})} \left\{ (3 - \alpha) \Phi' \cdot \Phi' + 2(\alpha + 1) V(\Phi) \right\} \\
& + (\alpha + 1) \left\{ 4 \oint \frac{W^{\alpha-1} \nabla_\mu \nabla^\mu F(\mathcal{R})^{2/3}}{F(\mathcal{R})^{2/3}} + \oint W^{\alpha+1} \left[\left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{(\nabla_a \nabla^a F(\mathcal{R})^{2/3})}{F(\mathcal{R})^{2/3}} \right] \right. \\
& - \frac{1}{6} \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_\mu F(\mathcal{R})^{2/3} \nabla^\mu F(\mathcal{R})^{2/3} + \frac{1}{6} \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3} \left. \right\} \\
& + \frac{4}{3} \alpha \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_\mu F(\mathcal{R})^{2/3} \nabla^\mu F(\mathcal{R})^{2/3} + \frac{4}{3} (3 - \alpha) \oint \frac{W^{\alpha+1}}{F(\mathcal{R})^{4/3}} \nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3}. \quad (24)
\end{aligned}$$

Among all consistency conditions encoded in (24), each related to a given α , there are many irrelevant. As a matter of fact, in order to explore the smooth branes possibility the choice $\alpha = -1$ is particularly elucidative, since it eliminates the overall warp factor. In fact, this choice provides simply

$$\oint \frac{\Phi' \cdot \Phi'}{F(\mathcal{R})} + \frac{1}{6\pi G_5} \oint \frac{\nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3}}{F(\mathcal{R})^{4/3}} = 0, \quad (25)$$

in which $\nabla_\mu F(\mathcal{R})^{2/3} = 0$ was already taken into account. Interestingly enough, Eq. (25) may be rewritten as

$$\oint \frac{\Phi' \cdot \Phi'}{F(\mathcal{R})} + \frac{1}{27\pi G_5} \oint (\ln |F(\mathcal{R})|)' \cdot (\ln |F(\mathcal{R})|)' = 0. \quad (26)$$

Now it turns out that whether $F(\mathcal{R})$ is positive, then it is impossible to achieve a smooth generalization of usual braneworld models, since the resulting constraint

$$\oint \left(\frac{1}{F(\mathcal{R})^{1/2}} \frac{d\Phi}{dr} \right) \cdot \left(\frac{1}{F(\mathcal{R})^{1/2}} \frac{d\Phi}{dr} \right) + \frac{1}{27\pi G_5} \oint (\ln |F(\mathcal{R})|)' \cdot (\ln |F(\mathcal{R})|)' = 0, \quad (27)$$

can never be satisfied. The situation is utterly different in the case of a negative $F(\mathcal{R})$. Obviously, in this last case the balance relation

$$\oint \left(\frac{1}{|F(\mathcal{R})|^{1/2}} \frac{d\Phi}{dr} \right) \cdot \left(\frac{1}{|F(\mathcal{R})|^{1/2}} \frac{d\Phi}{dr} \right) = \frac{1}{27\pi G_5} \oint (\ln |F(\mathcal{R})|)' \cdot (\ln |F(\mathcal{R})|)', \quad (28)$$

may be satisfied. Equation (28) perform, then, a clear criteria – $F(\mathcal{R} < 0)$ – for the possibility of smooth 3-branes in a five dimensional bulk. In comparing with the metric approach, where the negative quantity were proportional to $\oint F(\mathcal{R})^{-1} \nabla^2 F(\mathcal{R})$, the result obtained in the Palatini context is indeed exhaustive. As a final remark, notice that in the limit $F(\mathcal{R}) \rightarrow 1$, i. e. $f(\mathcal{R}) \rightarrow \mathcal{R}$, Eq. (28) reduces to $\oint \Phi' \cdot \Phi' = 0$, just as in usual General Relativity (as expected), a constraint which can never be reached.

V. CONCLUDING REMARKS

The modeling of warped smooth branes has given rise to a somewhat more formal branch of research in the context of braneworld gravity. In turn, this line of investigation has lead to the solidification of braneworld models in several different perspectives, since non-compact extra dimension [11], different bulk cosmological constants [12], and ingenious single thick branes approach [13], just to enumerate some. In which concern this paper, the general idea is not to set a specific model, but instead to provide a comprehensive scope from which consistent models can be constructed up.

It is shown that smooth generalizations of the usual Randall-Sundrum braneworld model can be achieved in $f(\mathcal{R})$ gravity. This is already know from previous work [8], but here we have worked in the Palatini formalism to $f(\mathcal{R})$. Apart from the fact that the metric and the Palatini formalisms to $f(\mathcal{R})$ are inequivalent, the analysis performed here has culminating into a more clear and assertive constraint to be fulfilled.

Even though it is not our purpose here the proposition of models, we shall emphasize that among all the possible generalizations leading to smooth 3-branes in a five dimensional within non-separable geometry (and a compact extra dimension), the use of $f(\mathcal{R})$ -Eintein-Palatini formalism seems to be a quite promising approach. This is because, once again, in this context it is possible to extract a simple and sharp necessary criteria.

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- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
 - [2] R. Maartens and K. Koyama, Living Rev. Relativity **13**, 5 (2010).
 - [3] M. Gremm, Phys. Lett. B **478**, 434 (2000).
 - [4] V. Dzhunushaliev, V. Folomeev, and M. Minamitsuji, Rept. Prog. Phys. **73**, 066901 (2010).
 - [5] P. Hořava and E. Witten, Nucl. Phys. B **460**, 506 (1996); P. Hořava and E. Witten, Nucl. Phys. B **475**, 94 (1996).
 - [6] G.W. Gibbons, R. Kallosh and A.D. Linde, J. High Energy Phys. **01**, 022 (2001).
 - [7] F. Leblond, R.C. Myers and D.J. Winters, J. High Energy Phys. **07**, 031 (2001).

- [8] G. P. de Brito, J. M. Hoff da Silva, P. Michel L. T. da Silva, and A. S. Dutra, *Int. J. Mod. Phys. D* **24**, 11 (2015).
- [9] V. Faraoni and N. Lanahan-Tremblay, *Phys. Rev. D* **77**, 108501 (2008).
- [10] V. Faraoni, *Class. Quantum Grav.* **26**, 145014 (2009).
- [11] A. Ahmed and B. Grzadowski, *J. High Energy Phys.* **01**, 177 (2013).
- [12] A. Ahmed, L. Dulny, and B. Grzadowski, *Acta Phys. Polon. B* **44** 11, 2381 (2013).
- [13] A. Ahmed, L. Dulny, and B. Grzadowski, *Eur. Phys. J. C* **74**, 2862 (2014).